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Localised Gravity in the Singular Domain Wall Background?

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ABSTRACT

We study singular, supersymmetric domain-wall solutions supported by the massive breathing mode scalars of, for example, sphere reductions in M-theory or string theory. The space-time on one side of such a wall is asymptotic to the Cauchy horizon of the anti-de Sitter (AdS) space-time. However, on the other side there is a naked singularity. The higher-dimensional embedding of these solutions has the novel interpretation of (sphere compactified) brane configurations in the domain “inside the horizon region,” with the singularity corresponding to the sphere shrinking to zero volume. The naked singularity is the source of an infinite attractive gravitational potential for the fluctuating modes. Nevertheless, the spectrum is bounded from below, continuous and positive definite, with the wave functions suppressed in the region close to the singularity. The massless bound state is formally excluded due to the boundary condition for the fluctuating mode wave functions at the naked singularity. However, a regularisation of the naked singularity, for example by effects of the order of the inverse string scale, in turn regularises the gravitational potential and allows for precisely one (massless) bound-state spin-2 fluctuating mode. We also contrast spectra in these domain wall backgrounds with those of the domain walls due to the massless modes of sphere reductions.

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1 Introduction

The study of domain walls in gauged supergravity theories has attracted much attention over the past year. (For a review on an earlier work, see [1].) In particular, since these solutions are asymptotic to anti-de Sitter (AdS) space-times, they are of importance in the study of the renormalisation group flows [2] of strongly coupled gauge theories in the context of the AdS/CFT correspondence. Typically these configurations are singular in the interior, and asymptotically approach the boundary of the AdS space-times.

On the other hand static AdS domain walls (in $D = 5$), which are asymptotic to AdS Cauchy horizons on either side of a non-singular wall, localise gravity on the wall [3, 4] (in $D = 4$), and thus have important phenomenological implications. It is therefore of great importance to find embeddings of gravity-trapping domain walls within a fundamental theory such as a compactified string or M-theory, a proposal initiated by H. Verlinde [5]. Of particular interest are field theoretic embeddings of such configurations in supergravity theories that arise as effective theories of string and M-theory compactifications. For example, scalar fields in the (abelian) vector supermultiplets of $N = 2$ gauged supergravity theories provide natural candidates for realising AdS domain walls. However, as it turns out these walls are in general singular in the interior, and on either side of the wall they are asymptotic to the boundary of AdS, as found for the one-scalar case in [6, 7] (and proven in [8, 9] for the multi-scalar case). The result is, essentially, a consequence of the fact that the gauged supergravity potential for scalars in vector supermultiplets has at most one non-singular supersymmetric extremum per non-singular domain, and this extremum is a *maximum*. (For a special choice of parameters for the gauged supergravity scalar potential, one side of the domain wall may asymptotically approach a non-AdS (“dilaton”) space-time [10].)

Recently, we investigated [11] domain-wall space-times supported by scalar fields belonging to massive supermultiplets. In particular, the breathing mode ϕ which parameterises the volume of the sphere, and which is a singlet under the isometry group of the sphere, provides a consistent truncation to a single massive scalar mode. Its potential has a supersymmetric AdS *minimum* (and not a maximum as in the case of massless vector supermultiplets) at $\phi = 0$. This allows for hybrid domain-wall solutions where on one side of the wall in the transverse direction $\rho < 0$ (in a co-moving coordinate), the space-time is asymptotic (as $\rho \rightarrow -\infty$) to the AdS horizon. The other side of the wall (the $\rho > 0$ region) allows for two possibilities:

- Branch I is a *singular* domain-wall solution with $\rho \rightarrow 0^+$ corresponding to a naked singularity at which $\phi \rightarrow \infty$ (zero volume of the sphere) [11].

- Branch II is a non-singular domain wall with, $\rho < 0$ corresponding to the dilatonic wall domain [12], and with $\rho \rightarrow \infty$ reaching the sphere decompactification ($\phi \rightarrow -\infty$). The higher-dimensional interpretation of this solution is a spherically compactified p -brane (in the $D = 5$ case, a D3-brane of Type IIB string theory compactified on the five-sphere S^5) in the domain that extends from the horizon to the asymptotically flat space-time. Unfortunately, this type of domain wall cannot trap gravity [11].

The purpose of this paper is to investigate the singular domain-wall solutions of Branch I. Owing to the naked singularity, such a solution is clearly undesirable from the classical point of view. However, quantum mechanics may be more tolerant of singularities, and we shall investigate the fluctuation spectrum in this singular background. Interestingly, the attractive gravitational potential is insufficiently singular at the naked singularity, thus rendering the spectrum bounded from below. In particular, the boundary condition on wave functions at the singularity allows for a continuous spectrum with positive energy, and the massless bound state is formally excluded. We also point out that a regularisation of the naked singularity, for example by modifying the metric at distances of the order of the inverse string scale, renders the gravitational potential finite and allows for precisely one (massless) bound-state of the lower-dimensional graviton. These issues are addressed in section 2.

In section 3 we contrast the spectrum in the background of such a singular breathing-mode domain-wall to those in domain-wall backgrounds supported by massless scalars of sphere reductions. These latter domain walls, which have been previously studied in the literature [13, 14, 15, 16, 17], also involve examples of singular solutions in the interior. However, on the AdS side they are asymptotic to the AdS boundary. In spite of the naked singularity, the fluctuation spectrum is well behaved. The spectrum of the fluctuating modes sheds light on the spectrum of strongly-coupled gauge theories via the AdS/CFT correspondence.

We also elucidate the nature of the higher-dimensional embedding of this configuration in Section 4; it describes a p -brane configuration in the domain “inside the horizon.” We show that the singularity structure of the massive scalar domain wall solution is exactly the same as the massless scalar domain wall associated with distributed p -branes with negative tension ingredients. This suggests that the inside of the horizon of non-dilatonic p -branes such as D3-branes or M-branes should be included in the discussion and the resolution of these singularities may be analogous to the one proposed in [18] for the singularity associated with negative tension.

2 Singular domain wall of the breathing mode

The scalar arising as the breathing mode in a Kaluza-Klein sphere reduction is massive, and in general yields a potential that allows a supersymmetric minimum with negative cosmological constant. The effective Lagrangian for the breathing-mode scalar and gravity is of the form:

$$\mathcal{L}_D = e R - \frac{1}{2} e (\partial\phi)^2 - e V, \quad (1)$$

where the potential is given by [19]

$$V = \frac{1}{2} g^2 \left(\frac{1}{a_1^2} e^{a_1\phi} - \frac{1}{a_1 a_2} e^{a_2\phi} \right). \quad (2)$$

The (positive) constants a_1 and a_2 are given by

$$a_1^2 = \frac{4}{k} + \frac{2(D-1)}{D-2}, \quad a_1 a_2 = \frac{2(D-1)}{D-2}, \quad (3)$$

where k is a certain positive integer. For $D = 4, 7$ and 5 , this integer takes the value $k = 1$. These cases correspond to the S^7 and S^4 reductions of $D = 11$ supergravity, and the S^5 reduction of type IIB supergravity, respectively. For $D = 3$ the integer k can be equal to 1, 2 or 3. The case $k = 1$ has a four-dimensional origin as an S^1 Scherk-Schwarz reduction of the Freedman-Schwarz model. The cases with $k = 2$ and $k = 3$ corresponding to S^3 and S^2 reduction of six-dimensional and five-dimensional supergravity theories.

The potential has one isolated AdS minimum $\Lambda \equiv V_{\min}$ at $\phi = 0$, tends to zero for $\phi \rightarrow -\infty$ (sphere decompactification) and tends to infinity for $\phi \rightarrow +\infty$ (zero volume of the sphere). The domain-wall solutions were obtained in [19]. In [11], the analytic solution in terms of a co-moving coordinate frame (simply related to the conformally flat metric) was derived, and the nature of the solutions was analysed in detail. They occur in two distinct branches, associated with two disconnected space-time regions. Here, we shall just describe the (asymptotic) behaviour of the solutions. (The analytical details can be found in [11].)

Using the co-moving coordinate ρ , the metric is of the form

$$ds^2 = e^{2A} dx^\mu dx_\mu + d\rho^2. \quad (4)$$

In the first branch, the coordinate ρ runs from $-\infty$ to 0, with

$$\begin{aligned} e^{2A} &\sim e^{2c\rho}, \quad \text{for } \rho \rightarrow -\infty, \\ e^{2A} &\sim \rho^\gamma, \quad \gamma = \frac{2a_2}{(D-2)a_1}, \quad \text{for } \rho \rightarrow 0. \end{aligned} \quad (5)$$

In the second branch, the coordinate ρ runs from $-\infty$ to $+\infty$, with

$$\begin{aligned} e^{2A} &\sim e^{2c\rho}, \quad \text{for } \rho \rightarrow -\infty, \\ e^{2A} &\sim \rho^\gamma, \quad \gamma = \frac{2a_1}{(D-2)a_2}, \quad \text{for } \rho \rightarrow +\infty. \end{aligned} \quad (6)$$

Thus we see that in both solutions, as $\rho \rightarrow -\infty$, the metric becomes AdS, with the constant c is given by $\Lambda = -(D-1)(D-2)c^2$. (Note that the Ricci tensor approaches $R_{\mu\nu} = -(D-1)c^2 g_{\mu\nu}$.) Clearly, the asymptotically AdS region $\rho \rightarrow \infty$ corresponds to the AdS Cauchy horizon. In the second branch, the solution is free of singularities, whilst in the first branch there is a naked singularity at $\rho = 0$. We shall concentrate on this first branch, and study the singular domain wall.

Close to the singularity point at $\rho = 0$, the values of the γ coefficient appearing in (5) for the various cases are summarised in Table 1.

	$D = 3$	$D = 4$	$D = 5$	$D = 7$
$k = 1$	$\frac{1}{2}$	$\frac{2}{7}$	$\frac{1}{5}$	$\frac{1}{8}$
$k = 2$	$\frac{2}{3}$			
$k = 3$	$\frac{3}{4}$			

Table 1: The values of the coefficient γ in (5) for the singular domain-wall solutions with the massive breathing mode, in various dimensions D .

2.1 Spectrum of fluctuating modes

It is of interest to examine the quantum fluctuations around the backgrounds of the Branch-I solutions. In particular, the spectrum may suffer from pathologies due to the singular nature of the metric.

The fluctuations of the D -dimensional graviton (in an appropriate gauge) are described by a minimally-coupled scalar field in this gravitational background [4]. The spectrum of these fluctuating modes in turn elucidates the nature of the modes in $(D-1)$ dimensions, and in particular the possibility of trapping a $(D-1)$ -dimensional massless graviton at such a domain wall. (For a detailed derivation of the equation for the fluctuating modes in arbitrary dimensions, see [20]). The minimally-coupled scalar field Φ obeys the wave equation

$$\partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0. \quad (7)$$

We make the Ansatz $\Phi = e^{ip \cdot x} \chi(z)$, where $m^2 = p \cdot p$ determines the mass of the fluctuating mode. It is helpful to cast the wave equation into the Schrödinger form, which can be done by first writing the metric in a manifestly conformally-flat frame, as

$$ds^2 = e^{2A(z)} (dx^\mu dx_\mu + dz^2), \quad (8)$$

by means of an appropriate coordinate transformation. For the Branch-I solutions that we are interested in, the coordinate z runs from $-\infty$ to 0, and $A(z)$ has the following asymptotic behaviour:

$$\begin{aligned} e^{2A} &\sim \frac{1}{c^2 z^2}, \quad \text{for } z \rightarrow -\infty, \\ e^{2A} &\sim z^{\tilde{\gamma}}, \quad \tilde{\gamma} = \frac{2\gamma}{2-\gamma}, \quad \text{for } z \rightarrow 0. \end{aligned} \quad (9)$$

Making the field redefinition $\chi = e^{-(D-2)A/2} \psi$, the wave equation assumes the form

$$(-\partial^2 - V) \psi = m^2 \psi, \quad (10)$$

with the Schrödinger potential given by

$$V = \frac{D-2}{2} A'' + \frac{(D-2)^2}{4} (A')^2. \quad (11)$$

The asymptotic behaviour of the potential is given by

$$\begin{aligned} V &\sim \frac{D(D-2)}{4z^2}, \quad \text{for } z \rightarrow -\infty, \\ V &\sim \frac{c}{z^2}, \quad \text{for } z \rightarrow 0. \end{aligned} \quad (12)$$

Thus we see that the potential near the singularity approaches a negative infinity, and there the value of the negative constant c is important. It is given by

$$c = -\frac{1}{4} + \frac{1}{(k+2)^2} > -\frac{1}{4}, \quad (13)$$

which is independent of the dimension D . The full form of the Schrödinger potential V is sketched in Figure 1.

Since we have $c > -\frac{1}{4}$, the spectrum is bounded from below. Namely, the boundary condition at $\rho = 0^-$ is $\Phi = 0$, thus disallowing solutions with $\Phi \neq 0$ at $\rho = 0^-$. As a consequence, the spectrum is continuous, with only positive energies occurring. Furthermore, the $m^2 = 0$ state has to be excluded, since it corresponds to the solution $\phi = \text{constant}$, which does not vanish at $\rho = 0$. The “bump” resulting from the AdS space-time causes the wave functions of the continuous spectrum to be suppressed in the interior region of the potential (close to the singularity). Thus although the background has a naked singularity, the spectrum for minimally-coupled scalar fields is well behaved. It does not, however, seem to be able to trap the massless $(D-1)$ -dimensional graviton.

The Schrödinger potential for the branch-II solution is quite different. It runs smoothly from $z = -\infty$ to $z = +\infty$, vanishing at both ends, with a single maximum at a certain finite value of z .

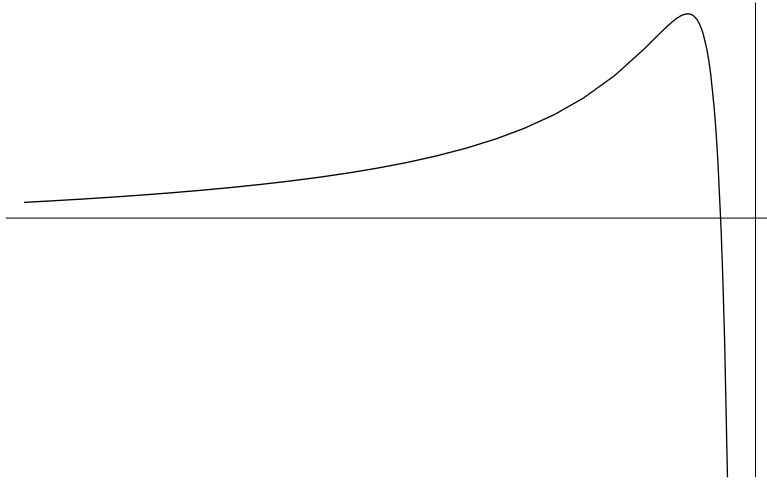


Figure 1: A sketch of the Schrödinger potential $V(z)$ as a function of z .

2.2 Regularising the metric near the naked singularity

It is on one hand encouraging that in spite of the naked singularity the spectrum is well behaved, but on the other hand it is disappointing that the massless mode is not bound (and is eliminated by the boundary condition at the singularity). However, this might just be an artefact of working only at the level of the effective supergravity theory. Thus it might be that string-induced corrections could “regulate” the metric near the naked singularity.

We shall show in the next section that the singularity structures of these massive-scalar domain-wall solutions are identical to those of the massless-scalar domain walls. The latter can be viewed in the higher dimension as continuous distributions of D3-branes or M-branes that include some negative-tension contributions. A resolution of singularities associated with negative-tension states was proposed in [18]. It is not inconceivable that a similar resolution can be applied to our cases, since the singularity structure is the same. As a consequence, the negative infinity of the Schrödinger potential could be cut off at distances $z \sim M_{string}^{-1}$ and thus in fact allow a zero-mass bound state after all. In the absence of a direct way of learning about these effects from string theory, here we present a model where we modify (*i.e.* regulate) the metric near the singularity in terms of a plausible, albeit somewhat ad-hoc, correction of order M_{string}^{-1} . This does at least provide an indication of the sort of modifications that can be expected once stringy corrections are taken into account.

Accordingly, near the singularity as $z \rightarrow 0^-$, and beyond $z > 0$, we shall modify the metric $A(z)$ so that it takes the form

$$e^{2A} = (|z| + M^{-1})^{\tilde{\gamma}}, \quad \text{for } z \sim \{-M^{-1}, 0^-\}, \quad (14)$$

$$e^{2A} = e^{-M'z} M^{-\tilde{\gamma}}, \quad \text{for } z > 0, \quad (15)$$

where M' and M are of the order of M_{string} . The positive coefficients $\tilde{\gamma}$ for the various examples are given by (9). Clearly, taking $\{M, M'\} \rightarrow \infty$ corresponds to the classical solution of the previous subsection. (The metric is continuous at $z = 0$, but its higher derivatives are not.)

Calculating the Schrödinger potential, we find that it now takes the following modified form:

$$V = \frac{c}{(|z| + M^{-1})^2}, \quad \text{for } z \sim \{-M^{-1}, 0^-\}, \quad (16)$$

$$V = \frac{1}{8}D(D-2)M'^2, \quad \text{for } z > 0. \quad (17)$$

Thus the negative infinity has been cut off with a step function, at distances $z \sim -M^{-1}$. (See the sketch of the modified potential in Figure 2).

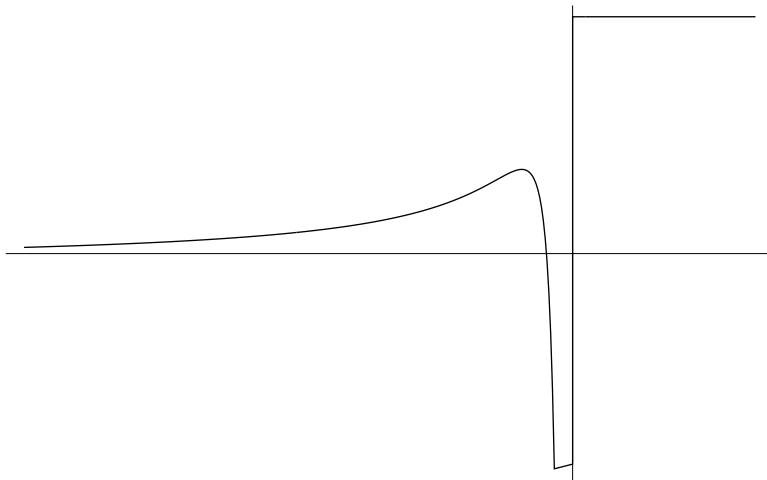


Figure 2: A sketch of the regularised Schrödinger potential $V(z)$ as a function of z . The negative infinity is cut-off at distances $z \sim -M_{string}^{-1}$ and at $z \geq 0$ a large barrier of order M_{string}^2 introduced.

After the regularisation of the metric, the potential has been rendered regular everywhere, and the Schrödinger equation can be written as a supersymmetric quantum mechanical system, which allows precisely one massless state (see for example [21]): $\psi = e^{(D-2)A(z)/2}$. From the asymptotic behavior of the metric one sees that in the “regulated” domain $z \geq 0^-$ the wave function falls off exponentially fast, with a decay constant of order M'^{-1} . On the other hand the fast fall off in the AdS regime $z \rightarrow -\infty$ renders the zero-mode renormalisable, and thus we have a *massless bound state* trapped on the wall.

3 Singular domain walls with “massless” scalars

Recently a large class of AdS domain-wall solutions, associated with diagonal symmetric potentials of lower-dimensional gauged supergravities, were obtained [14, 13, 15, 16, 17]. These solutions generally interpolate between a boundary AdS and a naked singularity, and thus it is of interest to revisit the properties of the fluctuation spectra here, and to contrast them with those of the domain walls supported by the massive breathing mode. The metrics for the solutions supported by the massless scalars are given by [16]:

$$\begin{aligned}
ds_D^2 &= (gr)^{\frac{4}{D-3}} \left(\prod_i H_i \right)^{\frac{1}{2} - \frac{2}{N}} dx^\mu dx_\mu + \left(\prod_i H_i \right)^{-\frac{2}{N}} \frac{dr^2}{g^2 r^2}, \\
H_i &= 1 + \frac{\ell_i^2}{r^2}, \quad N = \frac{4(D-2)}{D-3},
\end{aligned} \tag{18}$$

where $D = 4, 5$ and 7 , and for these dimensions we have $N = 8, 6$ and 5 respectively. The coordinate r runs from zero to infinity. At $r = \infty$ we have $H_i = 1$, and the metric describes AdS space-time in horospherical coordinates. Note that the AdS space-time is a maximum of the scalar potential.

As r approaches zero, the metric behaviour depends on the number of non-vanishing constants ℓ_i . If $n < N$ of them are non-vanishing, the metric approaches

$$\begin{aligned}
ds_D^2 &\sim \rho^\gamma dx^\mu dx_\mu + d\rho^2, \\
\gamma &= \frac{2(N-n)}{n(D-3)}, \quad \rho = r^{\frac{2n}{N}}.
\end{aligned} \tag{19}$$

The values of the constant γ for the various AdS domain walls are summarised in Table 2.

	$D = 4$	$D = 5$	$D = 7$
$n = 1$	14	5	2
$n = 2$	6	2	$\frac{3}{4}$
$n = 3$	$\frac{10}{3}$	1	$\frac{1}{3}$
$n = 4$	2	$\frac{1}{2}$	$\frac{1}{8}$
$n = 5$	$\frac{6}{5}$	$\frac{1}{5}$	
$n = 6$	$\frac{2}{3}$		
$n = 7$	$\frac{2}{7}$		

Table 2: The values of γ coefficients for the various domain walls supported by scalars of massless supermultiplets.

For all the cases, the metric has a power-law curvature singularity $\sim 1/\rho^2$ at $\rho = 0$. For $\gamma \geq 2$, the singularity is marginal, in the sense that $\rho = 0$ is also an horizon, whilst for $\gamma < 2$ the singularity at $\rho = 0$ is naked.¹ These solutions, when oxidised back to $D = 11$ or $D = 10$, become ellipsoidal distributions of M-branes or D3-branes. One can in general argue that the naked singularities of these lower-dimensional solutions are therefore artefacts of the dimensional reduction. However, in the case of the solutions with $n = N - 1$, the distributions involve negative-tension distributions [13, 16] of the M-branes or D3-branes, which clearly also have naked singularities in the higher dimension. It is interesting to note that for the cases associated negative tension distributions, the values of γ for $D = 4, 5$ and 7 are precisely the same as the ones for the massive-scalar breathing mode potentials given in Table 1.

It is therefore worthwhile to investigate whether quantum fluctuations around these backgrounds suffer from pathologies associated with the naked singularities. The minimally-coupled scalar Φ in these asymptotically AdS geometries is of special interest (in the dual gauge theory it corresponds to the operator that couples to the kinetic energy of the gauge field strength F^2). The spectrum for the wave equations of these background was studied in detail in various publications, and no pathologies in the spectrum were encountered in any of these cases [16, 17]. It is straightforward to show that the wave equation near the singularity $\rho = 0$ is then of the form

$$\left(-\partial_z^2 - \frac{C_n}{(z - z_*)^2} \right) \psi = m^2 \psi, \\ C_n = -\frac{1}{4} + \frac{(N - n - 2)^2}{(N - n - 4)^2}. \quad (20)$$

We see that the coefficients always satisfy the bound $C_n \geq -\frac{1}{4}$, which is essential in order for the energies of all the states to be bounded below.

It is of interest to investigate the relation between the structure of the spectrum and the nature of the curvature singularity of the background. It has been shown that the spectrum is discrete in the cases $N - n = 1, 2$ or 3 . For all of these values, the background suffers from a naked singularity. In fact, we can show that the spectrum is not only discrete, but also positive definite. This is because the coordinate r runs from $r = 0$ to $r = \infty$, and hence the wave function Φ has to satisfy the boundary conditions $\Phi(0) = 0$ and $\Phi(\infty)$ finite. It is straightforward to see that solutions with non-positive m^2 do not satisfy this condition. In particular, the massless solution, *i.e.* $\Phi = 1$, is excluded by the boundary conditions, since

¹There is an event horizon at $\rho = 0$ if geodesics originating at some finite and non-zero value of ρ take an infinite coordinate time to reach $\rho = 0$. Thus there is an horizon when $\gamma \geq 2$.

it does not vanish at $r = 0$. Indeed, the case of $D = 5$ and $n = 4$ can be solved exactly, and the spectrum can be seen to comprise only positive values of m^2 .

For $N - n \geq 4$, the singularity is marginally clothed by an event horizon, and the spectrum is continuous, (with a mass gap when $N - n = 4$). As we discussed earlier, the coordinate ρ terminates at $\rho = 0$ in all the cases, owing to the singularity, and it follows that the zero-mass solution has to be excluded from the spectrum since $\Phi(0)$ must vanish.

The absence of the $m^2 = 0$ states is consistent from the AdS/CFT viewpoint, since one does not expect gravity to be localised on the boundary of the AdS. Even in the case of a metric that is regulated at $\rho \rightarrow 0$, the zero mass state mode is *excluded*; this mode is of the form $\psi = e^{(D-2)A/2}$ (as obtained from the supersymmetric quantum mechanical analysis). However, this mode violates the boundary condition that $\psi = 0$ at the boundary of AdS.

4 Higher dimensional interpretation

The higher-dimensional interpretations of the domain-wall solutions supported by the massive breathing modes were given in [19]. They correspond to isotropic non-dilatonic branes in the higher dimensions, *e.g.*, M-branes, D3-branes and self-dual strings, *etc.* It is straightforward to see that the Branch-II solution corresponds to the region of the p -brane that interpolates between Minkowski space-time and the AdS throat. The Branch-I solution, on the other hand, corresponds to the region between the singularity (at zero volume of the sphere) and the horizon, as we shall demonstrate below.

It was shown in [22] that the D3-brane and M5-brane admit maximal analytic extensions that do not have any singularities. It is of interest therefore to examine the oxidation of the massive-scalar domain wall to ten or eleven dimensions in more detail, which we do by retracing the step of Kaluza-Klein reduction on the sphere. After doing this, the metric becomes [19]

$$\begin{aligned} ds_D^2 &= (\varepsilon H)^{2/(\hat{D}-d-1)} dx^\mu dx_\mu + H^{-2} d\rho^2 + \rho^2 d\Omega_d^2, \\ H &= 1 - \frac{Q}{\rho^{d-1}}, \end{aligned} \tag{21}$$

in terms of an appropriately-defined Schwarzschild-type coordinate ρ , where the constant ε is -1 for the Branch-I solution, and $+1$ for the Branch-II solution. (\hat{D} is the oxidation endpoint dimension.) Branch-I solution therefore provides a novel extension of p -brane solution into the interior of the horizon.

As was discussed in [22], when $\varepsilon = +1$ the exterior region of a metric such as (21) (*i.e.* the region $\rho > Q^{1/(d-1)}$ outside the horizon) can be smoothly extrapolated through

the horizon at $\rho = Q^{1/(d-1)}$ and out into another asymptotic region, thereby avoiding the curvature singularity at $\rho = 0$ altogether, provided that $\hat{D} - d - 1$ is even. This can be seen by defining a new radial coordinate w ,

$$w = H^{\frac{1}{\hat{D}-d-1}}, \quad (22)$$

in terms of which the metric (21) becomes

$$ds_{\hat{D}}^2 = w^2 dx^\mu dx_\mu + \kappa^2 \left(1 - w^{\hat{D}-d-1}\right)^{-\frac{2d}{d-1}} \frac{dw^2}{w^2} + Q^{\frac{2}{d-1}} \left(1 - w^{\hat{D}-d-1}\right)^{-\frac{2}{d-1}} d\Omega_d^2, \quad (23)$$

where $\kappa^2 = (\hat{D} - d - 1)^2 Q^{\frac{2}{d-1}} / (d - 1)^2$. Since ρ is an analytic function of w on the horizon at $w = 0$, one can analytically extend the metric to negative values of w , and so it is regular on the horizon. Furthermore, when $\hat{D} - d - 1$ is even, the metric is invariant under $w \rightarrow -w$, and so the extension to negative w is isometric to the original region with positive w [22]. Thus for the $D = 7$ Branch-II domain wall oxidised on S^4 to the M5-brane in $\hat{D} = 11$ (for which we have $\hat{D} - d - 1 = 11 - 4 - 1 = 6$), and the $D = 5$ Branch-II domain wall oxidised on S^5 to the D3-brane in $\hat{D} = 10$ (for which $\hat{D} - d - 1 = 10 - 5 - 1 = 4$), the metrics are completely non-singular. On the other hand the $D = 4$ Branch-II domain wall oxidises on S^7 to give an M2-brane in $\hat{D} = 11$, for which the singularity cannot be evaded by the $w \rightarrow -w$ reflection, since $\hat{D} - d - 1 = 11 - 7 - 1 = 3$.

In all cases the Branch-I solutions oxidise to the interior regions of the M-branes or D3-brane, and so the singularities remain in the higher dimension. The fact that the Branch-I solutions map into the interior regions of the higher-dimensional p -branes demonstrates that these interior regions do have a rôle to play, even in those cases where they are excised in the maximal analytic extension. From the lower-dimensional point of view, these naked singularities are no worse than the ones occurring in the domain-wall solutions associated with the Coulomb branches of the corresponding superconformal field theories on the AdS boundaries, as observed in section 3.

The cases $\gamma = \frac{2}{7}$, $\frac{1}{5}$ and $\frac{1}{8}$ can occur both from the massless-scalar potential and from the massive-scalar potential. For each dimension, these massless-scalar and massive-scalar solutions have in common that their higher-dimensional origins both involve naked singularities. In the massless-scalar case, the singularity is due to the negative tension of the distributed branes; in the massive-scalar case, the singularity is the one that is inside the horizon of the non-dilatonic p -brane. However in spite of the singularities, the minimally-coupled scalar field spectra are all well-behaved. This leads to an interesting question as to whether the interior region should be included in the discussion, even in the cases such as the M5-brane and D3-brane where it is normally excluded by making the maximal analytic

extension. A resolution of the singularity arising from a negative tension was proposed in [18]. A similar resolution may be applicable for the massive-scalar domain-wall solution too, since the singularity structure is the same.

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